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Wormhole Solution in Coupled Yang-Mills-Axion System

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Abstract

We show that wormhole solutions arise naturally in the effective action, resulting from a heterotic string theory, in which Einstein gravity is coupled to the antisymmetric tensor and a SU(2) Yang-Mills field. The Peccei-Quinn scale in this case gets related to the string tension which is natural in any string compactification.

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It is now recognized[1] that the effects of topology changes play an important role in quantum gravity. One of the consequences of such effects is the loss of quantum coherence as advocated by Hawking[2] and others[3,4] due to the nucleation of baby universes. Coleman[5], on the otherhand, has argued that the baby universes give rise to an indeterminancy in the constants of nature rather than to the loss of quantum coherence. Generalizing the earlier works of Hawking[6], it has then been argued that the probability distribution for the constants of nature are peaked like a delta-function at zero cosmological constant[7].

Hawking has explicitly constructed field configurations[8] which describe the creation of baby universes. These configurations, however, are not solutions of the field equations. Giddings and Strominger, on the other hand, have pointed out recently[9] that when Einstein gravity is coupled to the antisymmetric tensor field, there arise gravitational instanton solutions which induce topology changes in quantum gravity. The field strength of the antisymmetric tensor is related to the axion and, consequently, such solutions are also known as the axionic wormhole solutions. These solutions are extremely interesting in that they allow the action containing Euclidean gravity to be expanded around these nontrivial vacuua. This, in turn, makes it possible to compute the effects of topology changing processes in some detail and to test whether, in fact, an indeterminancy in the constants of Nature arises. It is interesting to note here that if the baby universes indeed introduce an indeterminancy in the coupling constants, then the hope of determining all constants from an underlying fundamental theory, such as the string theory, will be frustrated.

The string theory is expected to provide a consistent theory of quantum gravity. Therefore, it is meaningful to study the effects of the wormholes in an effective action based on the string theory. The wormholes solutions arising from such effective Lagrangians and the topology changing effects induced by them have recently been discussed[9,10]. Explicit wormhole solutions have also been constructed, recently, in the simplest of the coupled Yang-Mills-gravity system[11], namely, when the gauge group is SU(2). The purpose of this letter is to investigate whether wormhole solutions are possible in a more realistic effective action based on the string theory where both the Yang-Mills and the antisymmetric tensor fields are coupled to the Einstein gravity. The form of such an effective action, of course, can be determined from the action of a four dimensional heterotic string propagating in the background of the graviton, the antisymmetric tensor field and the Yang-Mills field[12]. But let us first recapitulate briefly the salient features of the axionic wormhole solution as well as the Yang-Mills wormhole solution.

The action describing the coupling of the antisymmetric tensor field to Euclidean gravity[4] has the form

$$S = \int d^4x\sqrt{g}(-\frac{1}{16\pi G}R + f^2 H_{\mu\nu\lambda}H^{\mu\nu\lambda}) \tag{1}$$

where the field strength for the antisymmetric tensor field is defined as

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu} \tag{2}$$

and is related to the axion field through

$$H_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda\rho}\partial^\rho\phi \tag{3}$$

The constant f in Eq.(1) is the Peccei-Quinn scale with the inverse dimension of length. Let us also note that in writing the action above, we have ignored a topological constant term as well as a surface term which do not influence the study of classical solutions. With the choice of a spherical metric

$$ds^2 = dt^2 + a^2(t)d^2\Omega_3 \tag{4}$$

it can be shown that the field configuration

$$H_{ijk} = \frac{n\epsilon_{ijk}}{f^2 a^3(t)} \tag{5}$$

satisfies the matter field equation

$$D^\mu H_{\mu\nu\lambda} = 0 \tag{6}$$

The evolution of a(t) is then determined from the Einstein equation. Let us note here that n in Eq.(5) is an integer which measures the axion charge.

In the case of a SU(2) Yang-Mills field coupled to gravity, the action with a cosmological constant is

$$S=\int d^4x\sqrt{g}\left(-\frac{1}{16\pi G}R-\frac{1}{4\beta^2}F^a_{\mu\nu}F^{\mu\nu,a}+\Lambda\right) \tag{7}$$

where β is the Yang-Mills coupling constant which has been scaled out so that

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+\epsilon^{abc}A^b_\mu A^c_\nu \qquad a,b,c=1,2,3 \tag{8}$$

and Λ is the cosmological constant. The matter field equation

$$\nabla_\mu F^{\mu\nu a}=0 \tag{9}$$

involves a covariant derivative which depends both on the metric as well as the Yang-Mills field. With a spherical ansatz for the metric[11], one can again show that the field configuration given by

$$\begin{aligned} F^a_{ij}e^{bi}e^{cj} &= -\frac{\epsilon^{abc}}{a^2(t)} \\ F^a_{4i}e^{bi} &= 0 \end{aligned} \tag{10}$$

with e^{ai}'s representing the three dimensional vierbeins, satisfy the matter field equation.

Let us next turn to the case where both the axion and the Yang-Mills field are coupled to Euclidean gravity with a cosmological constant. For simplicity, we again choose the gauge group to be SU(2). Let us recall that when a four dimensional heterotic string propagates in the background of the graviton, the antisymmetric tensor field and the Yang-Mills field, the presence of Weyl spinors gives rise to anomalies which render the theory inconsistent. Consistency requires that the antisymmetric tensor field, in such a case, transform nontrivially under a Yang-Mills transformation[14], namely,

$$\delta B_{\mu\nu}=\partial_{[\mu}\theta^a A^a_{\nu\;\;]} \tag{11}$$

Gauge invariance then forces the field strength $H_{\mu\nu\lambda}$ to include a Chern-Simons term, namely,

$$H_{\mu\nu\lambda} \rightarrow \tilde{H}_{\mu\nu\lambda} = H_{\mu\nu\lambda} + \frac{\alpha'}{8}\omega_{\mu\nu\lambda} \tag{12}$$

where

$$\begin{aligned} \omega_{\mu\nu\lambda} &= Tr A_{[\mu}F_{\nu\lambda]} = \frac{1}{3} \\ &= Tr\frac{1}{3}A_{[\mu}A_{\nu}A_{\lambda]} \end{aligned} \tag{13}$$

and α' is the string constant. The last part of eq. (13) is obtained using the relations between the Yang-Mills field strength and the potential which is a consequence of the ansate of ref. 11. The effective action for the coupled Yang-Mills-axion system inherits this additonal feature from the string theory and hence the action takes the form

$$\begin{aligned} S = \int d^4x\sqrt{g}(&-\frac{1}{16\pi G}R + f^2\tilde{H}_{\mu\nu\lambda}\tilde{H}^{\mu\nu\lambda} \\ &- \frac{1}{4\beta^2}F^a_{\mu\nu}F^{\mu\nu,a} + \Lambda) \end{aligned} \tag{14}$$

The axion and the Yang-Mills fields are now coupled through the Chern-Simons term and consequently, the ansatz for $H_{\mu\nu\lambda}$ and $F^a_{\mu\nu}$ given in Eqs. (5) and (10) respectively would no longer satisfy the matter field equations in general. In fact let us note that the equation of motion now take the form

$$\begin{aligned} D_\mu \tilde{H}^{\mu\nu\lambda} &= 0 \\ \nabla_\mu F^{a\mu\nu} &= 2\beta^2 f^2 \tilde{H}_{\nu\lambda\rho}F^{\lambda\rho,a} \\ G_{\mu\nu} &= 8\pi G T_{\mu\nu} \end{aligned} \tag{15}$$

where

$$\begin{aligned} T_{\mu\nu} = [2f^2(3\tilde{H}_{\mu\lambda\rho}\tilde{H}^{\lambda\rho}_{\nu} &- \frac{1}{2}g_{\mu\nu}(\tilde{H}_{\lambda\rho\sigma})^2) \\ &+ \frac{1}{\beta^2}(F^a_{\mu\nu}F^{\lambda,a}_{\nu} - \frac{1}{4}g_{\mu\nu}(F^a_{\lambda\rho})^2) \\ &- \Lambda g_{\mu\nu}] \end{aligned} \tag{16}$$

It is interesting to note that while the field configurations in Eqs. (5) and (10) do not satisfy the matter field equations of Eq. (15) in general; however, if

$$f^2 = \frac{6n}{\alpha'} \tag{17}$$

then

$$\tilde{H}_{\mu\nu\lambda} = 0 \tag{18}$$

and consequently, the matter field equations are automatically satisfied with a spherically symmetric metric ansatz of Hosoyu and Ugura[11] The time-time component of the Einstein equation, in the present case, becomes

$$(\frac{da(t)}{dt})^2 = 1 - \frac{4\pi G}{\beta^2 a^2(t)} - \frac{8\pi G\Lambda}{3} a^2(t) \tag{19}$$

This is precisely the equation obtained in the analysis of the Yang-Mills wormhole solution[11].

Thus our analysis shows that a realistic coupling of the Yang-Mills-axion system to Euclidean gravity based on the string theory does possess a wormhole solution if Eq. (17) is satisfied. Eq. (17) may seem unusual in that it requires the axion scale to proportional to planck mass. However, let us point out that in any compactification of the string such a result naturally arises[15]. (For comparison with the results of ref. 15, please note that the coupling constants can be restored by scaling $A^a_\mu \rightarrow \beta A^a_\mu$.) Thus, in fact, a condition such as in Eq. (17) is only natural and consequently, the wormhole solutions naturally arise in such a system. We may recall that such a high value for the axion scale is his couraging from the phenomenological point of view, but is an inherent feature of string theories.

To summarize, we have shown that wormhold solutions arise naturally in a realistic coupling of the Yang-mills-axion system to Euclidean gravity. Our results are derived within the context of SU(2). However, recently, a class of wormhole solutions have been obtained for the Einstein-Yang-Mills[16] system when the gauge group is SU(N) and it would be interesting to see whether our result would generalize to this case.

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